Taylor 级数

Guangyao Zhao

2022-12-18

Contents

Taylor 级数公式

$$y(x_{n+1}) = y(x_n) + \frac{h^1}{1!}y^{'}(x_n) + \frac{h^2}{2!}y^{''}(x_n) + \frac{h^3}{3!}y^{(3)}(x_n) + ...$$

- $y^{'} = \frac{dy}{dx} = f = f^{(0)}$
- $y^{''} = \frac{\partial f^{(0)}}{\partial x} + f\frac{\partial f^{(0)}}{\partial y} = f^{(1)}$
- $y^{(3)} = \frac{\partial f^{(1)}}{\partial x} + f\frac{\partial f^{(1)}}{\partial y} = f^{(2)}$
- 一般地说：$y^{j)} = \frac{\partial f^{(j-2)}}{\partial x} + f\frac{\partial f^{(j-2)}}{\partial y} = f^{(j-1)}$

具体地：

$$\begin{cases} y^{'} & = f \\ y^{''} & = \frac{\partial f}{\partial x} + f\frac{\partial f}{\partial y} \\ y^{(3)} & = \frac{\partial^2 f}{\partial x^2} + 2f\frac{\partial^2 f}{\partial x \partial y} + f^2\frac{\partial^2 f}{\partial y^2} + \frac{\partial f}{\partial y}\left(\frac{\partial f}{\partial x} + f\frac{\partial f}{\partial y}\right) \\ \vdots \end{cases}$$

Taylor 一阶表达式：

$$y_{n+1} = y_n + hy_n^{'}$$

截断误差

Taylor 一阶的截断误差：

$$y(x_{n+1}) - y_{n+1} = \frac{h^2}{2!} y^{(2)}(\zeta)$$

其中，$x_2 < \zeta < x_3$

定义：如果阶段误差为 $O(h^{p+1})$，则称该方法有 p 阶精度。一阶 Taylor 表达式有 1 阶精度。